

# DEVELOPPEMENTS LIMITES USUELS EN 0

Connaître les factorielles suivantes :  $3! = 6$ ,  $4! = 24$ ,  $5! = 120$ ,  $6! = 720$

**DL à un "petit ordre"**

$$\frac{1}{1-x} \underset{x \rightarrow 0}{=} 1 + x + x^2 + x^3 + o(x^3)$$

$$\frac{1}{1+x} \underset{x \rightarrow 0}{=} 1 - x + x^2 - x^3 + o(x^3)$$

$$e^x \underset{x \rightarrow 0}{=} 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)$$

$$\ln(1+x) \underset{x \rightarrow 0}{=} x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$\sin x \underset{x \rightarrow 0}{=} x - \frac{x^3}{6} + o(x^3) \quad (\text{ou } +o(x^4))$$

$$\sh x \underset{x \rightarrow 0}{=} x + \frac{x^3}{6} + o(x^3) \quad (\text{ou } +o(x^4))$$

**DL à un ordre quelconque**

$$\frac{1}{1-x} \underset{x \rightarrow 0}{=} 1 + x + x^2 + x^3 + \cdots + x^n + o(x^n)$$

$$\frac{1}{1+x} \underset{x \rightarrow 0}{=} 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + o(x^n)$$

$$e^x \underset{x \rightarrow 0}{=} 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + o(x^n)$$

$$\ln(1+x) \underset{x \rightarrow 0}{=} x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n+1} \frac{x^n}{n} + o(x^n)$$

$$\sin x \underset{x \rightarrow 0}{=} x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1})$$

$$\sh x \underset{x \rightarrow 0}{=} x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1})$$

**DL à un "petit ordre"**

$$\cos x \underset{x \rightarrow 0}{\equiv} 1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4) \quad (\text{ou } +o(x^5))$$

$$\operatorname{ch} x \underset{x \rightarrow 0}{\equiv} 1 + \frac{x^2}{2} + \frac{x^4}{24} + o(x^4) \quad (\text{ou } +o(x^5))$$

$$\tan x \underset{x \rightarrow 0}{\equiv} x + \frac{x^3}{3} + o(x^3) \quad (\text{ou } +o(x^4))$$

$$(1+x)^\alpha \underset{x \rightarrow 0}{\equiv} 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + o(x^2)$$

$$\sqrt{1+x} \underset{x \rightarrow 0}{\equiv} 1 + \frac{1}{2}x - \frac{1}{8}x^2 + o(x^2) \quad (\alpha = \tfrac{1}{2})$$

$$\operatorname{Arctan} x \underset{x \rightarrow 0}{\equiv} x - \frac{x^3}{3} + o(x^3) \quad (\text{ou } +o(x^4))$$

**DL à un ordre quelconque**

$$\cos x \underset{x \rightarrow 0}{=} 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n})$$

$$\operatorname{ch} x \underset{x \rightarrow 0}{=} 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n})$$

$$(1+x)^\alpha \underset{x \rightarrow 0}{=} 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \cdots + \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!} x^n + o(x^n)$$

$$\operatorname{Arctan} x \underset{x \rightarrow 0}{=} x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+1})$$