Entraînement au calcul algébrique : corrigé.

Correction de la question 1. **1°)** $\dfrac{1}{(-1)^n} = (-1)^n$

2°) $(-1)^{n+2} = (-1)^n(-1)^2 = (-1)^n$

3°) $(-1)^{2n} = \left((-1)^2\right)^n = 1^n = 1$

4°) $(-1)^{2n+1} = (-1)(-1)^{2n} = -1$ grâce à la question précédente

Correction de la question 2. **1°)**

$$\begin{aligned}(a+b)^2 &= a^2 + b^2 + 2ab\\ (a-b)^2 &= a^2 + b^2 - 2ab\\ (a+b)(a-b) &= a^2 - b^2\end{aligned}$$

2°)

$$\begin{aligned}(a+b+c)^2 &= (a+b+c)(a+b+c)\\ &= a^2 + ab + ac + ba + b^2 + bc + ca + cb + c^2\\ &= \boxed{a^2 + b^2 + c^2 + 2(ab + ac + bc)}\end{aligned}$$

Correction de la question 3. **1°)** $(a-b)(a^2+ab+b^2) = a^3 + a^2b + ab^2 - ba^2 - ab^2 - b^3 = \boxed{a^3 - b^3}$.

2°) $a^3 + b^3 = a^3 - (-b^3) = a^3 - (-b)^3 = (a - (-b))\left(a^2 + a(-b) + (-b)^2\right) = \boxed{(a+b)(a^2 - ab + b^2)}$.

3°) $27x^3 + 8 = 3^3x^3 + 2^3 = (3x)^3 + 2^3 = (3x+2)\left((3x)^2 - 3x \times 2 + 2^2\right) = \boxed{(3x+2)\left(9x^2 - 6x + 4\right)}$. Le discriminant du trinôme qui apparait est strictement négatif, on ne factorise pas plus dans $\mathbb{R}$.

Correction de la question 4. **1°)** $A = x^4 - x^2 = x^2x^2 - x^2 = x^2(x^2-1) = \boxed{x^2(x-1)(x+1)}$

2°)

$$\begin{aligned}B &= x^2 - 2x + 1 - (x-1)(2x+3)\\ &= (x-1)^2 - (x-1)(2x+3)\\ &= (x-1)(x - 1 - 2x - 3)\\ &= (x-1)(-x-4) = \boxed{(1-x)(x+4)}\end{aligned}$$

3°)

$$\begin{aligned}C &= (3x + 2 + x - 1)\,(3x + 2 - (x-1))\\ &= \boxed{(4x+1)(2x+3)}\end{aligned}$$

4°)

$$\begin{aligned} D &= x^2(x+1)+x+1 \\ &= \boxed{(x^2+1)(x+1)} \end{aligned}$$

5°)

$$\begin{aligned} E &= (3x)^2-7^2+(3x+7)(2x+3) \\ &= (3x+7)(3x-7)+(3x+7)(2x+3) \\ &= (3x+7)(3x-7+2x+3) \\ &= \boxed{(3x+7)(5x-4)} \end{aligned}$$

Correction de la question 5. **1°)** $\dfrac{7}{8}+\dfrac{5}{12}-\dfrac{4}{3}=\dfrac{7\times 3+5\times 2-4\times 8}{24}=-\dfrac{1}{24}$

2°) $\dfrac{1}{(n+1)^2}+\dfrac{1}{n+1}-\dfrac{1}{n}=\dfrac{n+n(n+1)-(n+1)^2}{n(n+1)^2}=-\dfrac{1}{n(n+1)^2}$

Correction de la question 6.

$x=\dfrac{\frac{a}{b}}{\frac{c}{d}}=\dfrac{a}{b}\dfrac{1}{\frac{c}{d}}=\dfrac{a}{b}\dfrac{d}{c}=\boxed{\dfrac{ad}{bc}}.$

$y=\dfrac{a}{\dfrac{b}{\frac{c}{d}}}=\dfrac{a}{\dfrac{bd}{c}}=\boxed{\dfrac{ac}{bd}}.$

$z=\dfrac{\dfrac{a}{\frac{b}{c}}}{d}=\dfrac{\dfrac{ac}{b}}{d}=\boxed{\dfrac{ac}{bd}}.$

$t=\dfrac{\dfrac{\frac{a}{b}}{c}}{d}=\dfrac{\dfrac{a}{bc}}{d}=\boxed{\dfrac{a}{bcd}}.$

$A=\dfrac{\frac{a^2}{3b}}{\frac{ac}{6b}}=\dfrac{6ba^2}{3bac}=\boxed{\dfrac{2a}{c}}.$

$B=\dfrac{\dfrac{3}{10}\times\dfrac{15}{\frac{9}{2}}}{\dfrac{9}{15}\times\dfrac{5}{2}}=\dfrac{\dfrac{3}{5\times 2}\times\dfrac{5\times 3}{3\times 3}\times 2}{\dfrac{3\times 3}{3\times 5}\times\dfrac{5}{2}}=\dfrac{1}{\frac{3}{2}}=\boxed{\dfrac{2}{3}}.$

$C=\dfrac{6\left(3-\frac{1}{2}\right)\left(4+\frac{1}{3}\right)}{12\left(5+\frac{1}{4}\right)\left(7-\frac{1}{3}\right)}=\dfrac{\frac{5}{2}\frac{13}{3}}{2\frac{21}{4}\frac{20}{3}}=\dfrac{\dfrac{5\times 13}{2\times 3}}{\dfrac{21\times 4\times 5}{2\times 3}}=\dfrac{13}{21\times 4}=\boxed{\dfrac{13}{84}}.$

Correction de la question 7. $A = \dfrac{x+1}{(x-1)(x+1)} - \dfrac{x-1}{(x+1)(x-1)} + \dfrac{2x}{1-x^2}$
$= \dfrac{x+1-x+1}{x^2-1} - \dfrac{2x}{x^2-1} = \dfrac{2-2x}{x^2-1} = \dfrac{-2(x-1)}{(x-1)(x+1)} = \dfrac{-2}{x+1}$.

$$B = \frac{b(x-a)}{abx} + \frac{x(a-b)}{abx} + \frac{a(b-x)}{axb} = \frac{bx-ba+xa-xb+ab-ax}{abx} = 0$$

Correction de la question 8. $\dfrac{14^2 \times 9^2}{3^5 \times 7} = \dfrac{7^2 \times 2^2 \times \left(3^2\right)^2}{3^5 \times 7} = \dfrac{7 \times 4 \times 3^4}{3^5} = \dfrac{7 \times 4}{3} = \boxed{\dfrac{28}{3}}$

Correction de la question 9. $x = \dfrac{2^3 3^2}{3^{-2}2^{-2}3^4 2^8} = \dfrac{2^3 3^2}{3^2 2^6} = \boxed{2^{-3}}$
$y = 2^{100} + 2^{101} = 2^{100}(1+2) = \boxed{3.2^{100}}$
$z = 2^{101} - 2^{100} = 2^{100}(2-1) = \boxed{2^{100}}$
$t = \boxed{2.3^{15}}$
$u = \dfrac{\left(3^2(-2)^4\right)^8}{\left((-3)^5 2^3\right)^{-2}} = 3^{16}2^{32}\left((-3)^5 2^3\right)^2 = 3^{16}2^{32}3^{10}2^6 = \boxed{2^{38}3^{26}}$

Correction de la question 10.

$A = 3x^2y^3 - y(xy)^2 = 3x^2y^3 - yx^2y^2 = 3x^2y^3 - x^2y^3 = \boxed{2x^2y^3}$.

$B = \dfrac{4x^2y^3 - (xy)^2y}{x^2y^2 \times (-x)^3} = \dfrac{4x^2y^3 - x^2y^3}{-x^2y^2x^3} = -\dfrac{3x^2y^3}{x^5y^2} = -\dfrac{3x^2y^2 \times y}{x^2y^2 \times x^3} = \boxed{-\dfrac{3y}{x^3}}$.

$C = \dfrac{(-a)^7 \times (-b^2c^3)^3}{-b^3c \times (-a)^5} = \dfrac{(-a^7) \times (-b^6c^9)}{-b^3c \times (-a^5)} = \dfrac{a^7b^6c^9}{a^5b^3c} = \boxed{a^2b^3c^8}$.

Correction de la question 11. $A = \dfrac{4}{3-\sqrt{5}} = \dfrac{4(3+\sqrt{5})}{9-5} = \boxed{3+\sqrt{5}}$

$$B = \frac{1}{\sqrt{\sqrt{2}}}\sqrt{\frac{1+\sqrt{2}}{2}} = \sqrt{\frac{1+\sqrt{2}}{2\sqrt{2}}} = \sqrt{\frac{\sqrt{2}+2}{4}} = \boxed{\frac{\sqrt{\sqrt{2}+2}}{2}}$$

$$C = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} - \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{3-2} - \frac{(\sqrt{3}-\sqrt{2})^2}{3-2}$$
$$= 3+2+2\sqrt{6} - (3+2-2\sqrt{6}) = \boxed{4\sqrt{6}}$$

Correction de la question 12.
$$A = \frac{\dfrac{1}{1+x} + \dfrac{1-x}{(1+x)^2}}{\sqrt{\dfrac{1-x}{1+x}}\left(1+\dfrac{1-x}{1+x}\right)} = \frac{\dfrac{1+x+1-x}{(1+x)^2}}{\dfrac{\sqrt{1-x}}{\sqrt{1+x}}\dfrac{1+x+1-x}{1+x}} = \frac{2}{(1+x)^2}\frac{1}{\dfrac{2\sqrt{1-x}}{(1+x)^{\frac{3}{2}}}} = \frac{1}{(1+x)^{\frac{1}{2}}}\frac{1}{\sqrt{1-x}}$$
$$A = \frac{1}{\sqrt{(1+x)(1-x)}} = \boxed{\frac{1}{\sqrt{1-x^2}}}$$

Correction de la question 13. Pour $A = \dfrac{\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{2}}}}{\dfrac{1}{1+\dfrac{1}{1-\dfrac{1}{2}}}}$:

$$1+\frac{1}{1+\frac{1}{2}} = 1+\frac{1}{\frac{3}{2}} = 1+\frac{2}{3} = \frac{5}{3} \quad \text{et} \quad 1+\frac{1}{1-\frac{1}{2}} = 1+\frac{1}{\frac{1}{2}} = 1+2 = 3.$$

Donc $A = \dfrac{\frac{3}{5}}{\frac{1}{3}} = \boxed{\dfrac{9}{5}}$.

$$B = \frac{2+\frac{2+a}{2-a}}{2-\frac{2+a}{2-a}} = \frac{\frac{2(2-a)+2+a}{2-a}}{\frac{2(2-a)-2-a}{2-a}} = \frac{4-2a+2+a}{4-2a-2-a} = \boxed{\frac{6-a}{2-3a}}$$

Correction de la question 14.

$$\begin{aligned} A &= \frac{\frac{b+a-x}{ab}(x+a+b)}{\frac{b^2+a^2+2ab-x^2}{a^2b^2}} \\ &= \frac{(b+a-x)(x+a+b)}{b^2+a^2+2ab-x^2}ab \\ &= ab \end{aligned}$$

$$\begin{aligned} B &= \frac{(2x+3)^2}{2(2x-3)(2x+3)} - \frac{24x}{2(2x-3)(2x+3)} + \frac{(3-2x)(2x-3)}{2(2x+3)(2x-3)} \\ &= \frac{(2x+3)^2-24x-(2x-3)^2}{2(2x-3)(2x+3)} \\ &= \frac{(2x+3-(2x-3))\,(2x+3+2x-3)-24x}{2(2x-3)(2x+3)} \\ &= \frac{6.4x-24x}{2(2x-3)(2x+3)} \\ &= 0 \end{aligned}$$

Correction de la question 15. $7840 = 10\times 784 = 2\times 5\times 2\times(350+42) = 2^2\times 5\times 2\times(175+21) = 2^3\times 5\times 196 = 2^3\times 5\times 2\times 98 = 2^4\times 5\times 2\times 49 = \boxed{2^5\times 5\times 7^2}$

Correction de la question 16. $A=\dfrac{-3\left(\frac{2}{3}\right)^2+8\left(\frac{7}{2}\right)^2}{5\left(\frac{2}{5}\right)^2-6\left(\frac{4}{3}\right)^2}=\dfrac{-\dfrac{4}{3}+2\times 7^2}{\dfrac{4}{5}-2\dfrac{16}{3}}=\dfrac{\dfrac{2}{3}(-2+3\times 49)}{\dfrac{4}{15}(3-5\times 8)}=\dfrac{\dfrac{2}{3}\times 145}{\dfrac{4}{15}\times(-37)}=$ $-\dfrac{2\times 145\times 15}{4\times 3\times 37}=-\dfrac{145\times 5}{2\times 37}=\boxed{-\dfrac{725}{74}}$

Correction de la question 17.
$12^3\times 3^3=(12\times 3)^3=\boxed{36^3}$

$125^2\times 3^6=\left(5^3\right)^2\times 3^6=5^6\times 3^6=\boxed{15^6}$

$3^3\times 5^6=3^3\times\left(5^2\right)^3=(3\times 25)^3=\boxed{75^3}$.

$7^2\times 2^3$ ne peut pas s'écrire sous la forme voulue.

Correction de la question 19. **1°)**

$$\begin{aligned}A&=16(2x+7)^2-25(3x-7)^2=(4(2x+7))^2-(5(3x-7))^2\\&=(4(2x+7)-5(3x-7))\,(4(2x+7)+5(3x-7))\\&=(-7x+63)(23x-7)=\boxed{7(9-x)(23x-7)}\end{aligned}$$

2°)

$$\begin{aligned}B&=9x^2(2x+1)-(2x+1)\\&=(9x^2-1)(2x+1)\\&=\left((3x)^2-1\right)(2x+1)\\&=\boxed{(3x-1)(3x+1)(2x+1)}\end{aligned}$$

3°)

$$\begin{aligned}C&=(4x^2-25)(x+2)-(x^2-4)(2x+5)+(5x+10)(2x+5)\\&=(2x-5)(2x+5)(x+2)-(x-2)(x+2)(2x+5)+5(x+2)(2x+5)\\&=(2x+5)(x+2)\,(2x-5-(x-2)+5)\\&=(2x+5)(x+2)(x+2)=\boxed{(2x+5)(x+2)^2}\end{aligned}$$

Correction de la question 20. Comme 4 et 5 sont strictement positifs, $\dfrac{7}{4}<\dfrac{9}{5}\iff 7\times 5<9\times 4$ ie $35<36$, ce qui est bien le cas donc on a bien : $\dfrac{7}{4}<\dfrac{9}{5}$.

Correction de la question 21. Comme $1+\sqrt{2}$ et $\sqrt{3}$ sont positifs, il suffit de comparer leurs carrés. $(1+\sqrt{2})^2=1+2+2\sqrt{2}=3+2\sqrt{2}>\sqrt{3}^2$. Donc, $1+\sqrt{2}>\sqrt{3}$.

Correction de la question 22. Calculons $\left(\frac{1+\sqrt{5}}{4}\right)^2 = \frac{1+5+2\sqrt{5}}{16} = \frac{3+\sqrt{5}}{8}$.

Comme $\frac{1+\sqrt{5}}{4}$ est un nombre positif, on a bien $\sqrt{\frac{3+\sqrt{5}}{8}} = \frac{1+\sqrt{5}}{4}$.

Correction de la question 23. Justifions d'abord que A existe.

$(4\sqrt{3})^2 = 48 < 7^2 = 49$ donc $4\sqrt{3} < 7$ donc $7 - 4\sqrt{3} > 0$.

Calculons A^2 pour commencer.

$$\begin{aligned} A^2 &= \left(\sqrt{7-4\sqrt{3}} - \sqrt{7+4\sqrt{3}}\right)^2 \\ &= 7 - 4\sqrt{3} + 7 + 4\sqrt{3} - 2\sqrt{7-4\sqrt{3}}\sqrt{7+4\sqrt{3}} \\ &= 14 - 2\sqrt{(7-4\sqrt{3})(7+4\sqrt{3})} = 14 - 2\sqrt{49-48} = \boxed{12} \end{aligned}$$

$0 \leq 7 - 4\sqrt{3} < 7 + 4\sqrt{3}$ donc $\sqrt{7-4\sqrt{3}} < \sqrt{7+4\sqrt{3}}$. Ainsi, $A < 0$.

On en déduit que : $\boxed{A = -\sqrt{12} = -2\sqrt{3}}$.

Correction de la question 24. $e^{3\ln 2} = 2^3 = 8$.

Correction de la question 25. Cette expression existe sur $\mathbb{R}^*$.

$\forall x \in \mathbb{R}^*, \ln(x^2) = \ln(|x|^2) = 2\ln(|x|)$ car $|x| > 0$.

Correction de la question 26.

$$\begin{aligned} 2\ln\left(\frac{3}{4}\right) - 3\ln\left(\frac{3}{8}\right) &= 2\ln(3) - 2\ln(4) - 3\ln(3) + 3\ln(8) \\ &= -\ln(3) - 2\ln(2^2) + 3\ln(2^3) \\ &= -\ln(3) - 2.2\ln(2) + 3.3\ln(2) \\ &= 5\ln(2) - \ln(3) \\ \ln\left(\frac{1}{\sqrt{2}}\right) &= -\ln\left(\sqrt{2}\right) \\ &= -\frac{1}{2}\ln(2) \end{aligned}$$

Correction de la question 27. L'expression existe ssi $x > -1$.

$e^{x-\ln(x+1)} = e^x e^{-\ln(x+1)} = e^x \frac{1}{e^{\ln(x+1)}} = \frac{e^x}{x+1}$.